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# Influence of double seasonality on economic forecasts on the example of energy demand

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**Abstract.** The article deals with the issues of forecasting one of the most important elements of modern economies, which is electricity. The size of its production must be properly predicted and it should be technically possible and profitable from the economic point of view. The most frequently used forecasting models in literature are ARIMA and exponential smoothing. However, these models have a major disadvantage as they do not allow forecasting the series with double seasonality and periods of non-integer values. The authors of this article, on the example of data on maximum energy demand, presented an application of one of the latest forecasting models TBATS, which is devoid of this disadvantage. Using this model, the article presents forecasts for the year ahead with information on power consumption for each day, showing that long-term planning without losing details is possible. With such information energy producers are able to optimize production and the economic side of their business overall.

**Keywords:** electricity demand, forecasting time series with seasonality, TBATS, Fourier series, ARIMA, exponential smoothing.

JEL Classifications: Q47, Q41, O13, N70

### INTRODUCTION

Electricity is the basis for any today's economy functioning. Electricity customers regardless whether they are households, businesses or institutions are interested in the continuity of its supply, and the issues of production and supply size rest on power plants. This obvious fact is not so simple to implement, because it

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DOI: 10.14254/2071-8330.2016/9-3/1 does not cover technical and economic factors. Energy producers have to forecast future energy demand so that its production is enough to meet the needs of customers, and at the same time not to produce its excessively, because collection of electricity is technically problematic and economically unviable. The most commonly used forecasting methods in literature is the classic family of ARIMA models introduced by Box and Jenkins (Box & Jenkins, 1976) and the models of exponential smoothing with the modifications introduced by Holt (Holt, 1957), or in the recent years also by Hyndman (Hyndman, et al., 2002). These modifications aim at better matching of models for various types of data, taking into account seasonal variations, trends and seasonality. Despite the existence of many models for forecasting time series, there are still the new ones which take into account the specific types of data and give more accurate forecasts than the previously known models. An example of a peculiar type of data, which also require special models is the demand for electricity. The data of this type irrespective of country have similar characteristics such as overlapping cycles of higher and lower power requirements dependent on both days of the week and on the month. Nowadays, when the world cannot function without electricity as such, correct forecasting of power demand is very important for power systems. Forecasts of this kind are important not only to ensure the right amount of power for customers, but also for the optimal load of power systems and to avoid overproduction of power whose storage is problematic and expensive. Using the most popular models which take into account seasonality, it is possible to predict the data from one seasonality for several periods ahead. For example, having a time series that contains monthly data for power demand and applying the SARIMA model, it is possible to forecast correctly for several months, as it was presented in the previous article (Brożyna, et al., 2016). With daily data and using the same model it is still possible to forecast for several periods, but this time these are only days. Thus, traditional models allow forecasting general data for a few months ahead or more detailed data, but for a much shorter period, only a dozen days. In situations where traditional models are no longer sufficient, the new ones arise which take into account the problematic issues. An example of such a model is a Trigonometric, Box-Cox transform, ARMA errors, Trend, and Seasonal components model (TBATS) which can include both annual seasonality and the daily one, so that forecast may be determined for few months, and at the same time may contain data on power demand for each day. Using this particular model, the paper presents its application on the example of daily data on electricity demand in Poland. The research part of the work is divided into three parts. The first contains the description of daily data on the demand for electricity. The second part describes the TBATS model and the literature related to this issue. In the last part of the research, by using the previously submitted data and model predictions, medium of particulars as to the short-term forecasts are presented. The study also explains the advantages from using the tested here model.

#### DATA CHARACTERISTICS

In this study we have performed a forecast with double seasonality for an electricity demand in the Polish National Power System (NPS). For this purpose we have used quarter-hourly data (measured in megawatts [MW]), obtained from the website of the Polish Power System<sup>1</sup>. The full sample covers the period from January 2002 up to the end of October 2015, which gives a time series with a length of nearly a half a million observations.

In order to present issues concerning forecasting time series with double seasonality, the quarter-hourly data were aggregated into a daily data, by selecting the maximum value from a given period. Descriptive statistics for a final sample, which contains 5052 observations expressed in gigawatts [GW], are presented in Table 1.

<sup>&</sup>lt;sup>1</sup> Polskie Sieci Elektroenergetyczne, http://www.pse.pl/index.php?dzid=77 (02.11.2015)

N	Minimum	1st Quarter	Median	Mean	3rd Quarter	Maximum
	[GW]	[GW]	[GW]	[GW]	[GW]	[GW]
5052	12.75	18.28	20.14	19.99	21.97	25.84

Table 1. Descriptive statistics of the maximum power demand
in the National Power System in Poland [GW]

Source: Own study.

We can see a significant difference between minimum and maximum power demand in the period considered, reaching 13 GW. At the same time the value of interquartile range is less than 4 GW, which shows the average variability in the analyzed series.

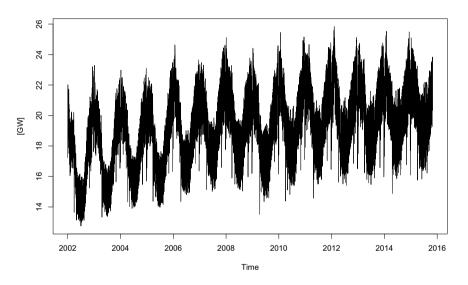


Figure 1. Daily maximum power demand in National Power System in Poland Source: Own study.

While analysing daily maximum power demand in the Polish National Power System (Figure 1), we can observe a continuous increase in energy demand. Between 2008 and 2010 we can see a change in the trend. This may be a consequence of the global financial crisis that began in 2007 (Goodhart, 2008); (Nieborak, 2010); (Zwolankowski, 2011). The Polish economy was negatively affected by the crisis (Nazarczuk, 2013), which ultimately translates into the decrease in energy consumption in various sectors (Platchkov & Pollitt, 2011; Balitskiy et al., 2014, 2016; Mentel, 2012).

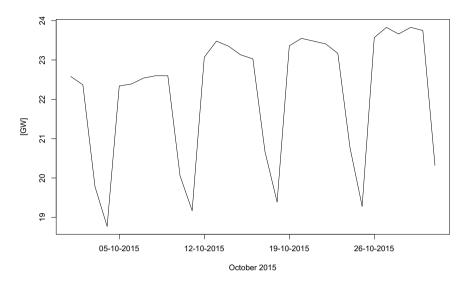


Figure 2. Daily, maximum power demand in National Power System in Poland in October 2015 Source: Own study.

Additionally we can notice that the analyzed series is characterized by an annual seasonality. On the enlargement of the figure 1 piece for October 2015 (Figure 2), we can also observe a weekly seasonality with a clear, greater demand for power within 5 working days and a significant decline in demand at weekends. Weekly fluctuations are directly related to the functioning of the Polish economy in which manufacturing plants and offices work in the vast majority from Monday to Friday, and Saturday and Sunday is the traditional day-off for employees. Thus, the decline in energy demand in the weekend is a result of lower power consumption by the industry.

## MODEL

Based on Figure 1 and Figure 2 we can see that the series have a visible growing trend and a double seasonality: the annual and the weekly one. Presence of the double seasonality limits the choice of the possible forecasting method. The models often used in the literature such as ARIMA (Pappas, et al., 2008); (Chen, et al., 1995); (Lee & Ko, 2011); (de Andrade & da Silva, 2009), exponential smoothing (Taylor, 2003); (Hyndman, et al., 2008) or the method of homonymous trends (Zeliaś, et al., 2016); (Bee Dagum & Bianconcini, 2016) were designed to forecast the time series which contain small amounts of data with at most one seasonality and cannot be used for the case where the data includes overlapping annual and weekly seasonality. In addition, even if these models take into account seasonality, the considered period must be an integer. Fractions such as 365.25 days or 52.18 weeks per year, which is common for years, when February has 29 days, are not permitted. To forecast this type of data the TBATS model (Trigonometric, **B**ox-Cox transform, **A**RMA errors, **T**rend, and **S**easonal components) was developed a few years ago (De Livera, et al., 2011).

This model has the following arguments:

TBATS( $\omega$ , {p, q},  $\varphi$ , {<m<sub>1</sub>, k<sub>1</sub>>, <m<sub>2</sub>, k<sub>2</sub>>,...,<m<sub>T</sub>, k<sub>T</sub>>}) where:

 $\omega$  is a Box-Cox transformation (Box & Cox, 1964),

*p*, *q* are ARMA parameters (Whittle, 1951); (Box & Jenkins, 1976); (Brockwell & Davis, 1996), φ is a damping parameter (Gardner & McKenzie, 1985); (Snyder, 2006),

 $m_1, \dots, m_T$  are seasonal periods,

 $k_{1},...,k_{T}$  are number of Fourier series pairs (West & Harrison, 1997); (Harvey, 1989).

The model can be written as:

$$y_{t}^{(\omega)} = l_{t-1} + \varphi b_{t-1} + \sum_{i=1}^{T} s_{t-1}^{(i)} + \alpha d_{t}$$
$$b_{t} = b_{t-1} + \beta d_{t}$$
$$s_{t}^{(i)} = \sum_{j=1}^{k_{t}} s_{j,t}^{(i)}$$
$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_{j}^{(i)} + s_{j,t-1}^{*} \sin \lambda_{j}^{(i)} + \gamma_{1}^{(i)} d_{t}$$
$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_{j}^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_{j}^{(i)} + \gamma_{2}^{(i)} d_{t}$$
$$\lambda_{j}^{(i)} = \frac{2\pi j}{m_{i}}$$

where:

*i*=1,...,*T*   $d_i$  is an ARMA(*p*,*q*) process,  $\alpha$ ,  $\beta$ ,  $\gamma_i$  and  $\gamma_2$  are smoothing parameters,  $\ell_o$  is initial level,  $b_o$  is slope value.

To estimate the forecasting errors of this model we have used the most popular measures proposed by (Bratu, 2012), such as:

Mean Absolute Error 
$$MAE = \frac{\sum_{i=1}^{n} |e_i|}{n}$$
  
Mean Squared Error  $MSE = \frac{\sum_{i=1}^{n} e_i^2}{n-1}$ 

**R**oot Mean Squared Error  $RMSE = \sqrt{MSE}$ 

Mean Absolute Percentage Error 
$$MAPE = \frac{\sum_{t=1}^{n} \left| \frac{e_t}{y_t} \right|}{n} \cdot 100\%$$

Following (Hyndman & Koehler, 2006) and (Gardner, 1985) we have also applied some less popular forecasting error measures, such as:

Mean Error 
$$ME = \frac{\sum_{i=1}^{n} e_i}{n}$$
  
Mean Percentage Error  $MPE = \frac{\sum_{i=1}^{n} \frac{e_i}{y_i}}{n} \cdot 100\%$   
Mean Absolute Scaled Error  $MASE = \frac{\sum_{i=1}^{n} |e_i|}{\frac{n}{n-m} \sum_{i=m+1}^{n} |y_i - y_{i-m}|}$   
Autocorrelation function of errors at lag 1  $ACF1 = \frac{\sum_{i=1}^{n-1} (e_i - ME) \cdot (e_{i+1} - ME)}{\sum_{i=1}^{n} (e_i - ME)^2}$ 

where:

 $e_t$  is an error term  $e_t = y_t - y_t^*$  $y_t$  is the actual representation of a value in the period t,  $y_t^*$  is the forecasted value in the period t, *m* is the number of seasons (Hyndman & Koehler, 2006).

Comparing the ME and MAE errors or MPE and MAPE we receive information about whether the values in the forecast are systematically lower or higher than the values observed, or are they multidirectional. The analysis of mean error (MSE) may, however, point out errors which are unusually large. Significant differences between MAE and RMSE indicate errors with extremely large values.

# FORECAST OF DAILY DATA

The analyzed time series contain daily data and are characterized by the annual seasonality (season 2) and the weekly seasonality (season 1). The results of the decomposition (Figure 3) confirm the presence of an annual seasonality (season 2). The frequency of the first seasonal component (season 1) in relation to the total length of the considered time series, makes that its identification is not possible based upon the chart.

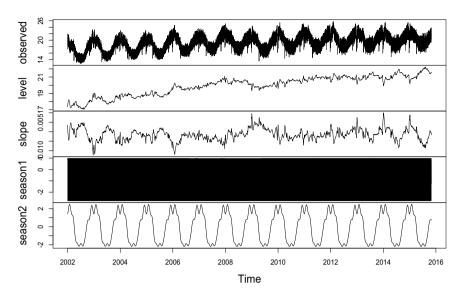


Figure 3. Decomposition of the time series with data of daily power demand of NPS Source: Own study.

Only after the decomposition of season 1 for a shorter period, e.g. one month, as it has been shown in Figure 4, we were able to correctly identify a weekly cycle pattern. It can be noticed that the weekly seasonality shows similar behaviour to the annual seasonality identified in Figure 2, i.e. the demand for energy in a weekly cycle is higher (and at a similar level) at working days from Monday to Friday, and lower on Saturdays and Sundays, when most employees have days off, and thus the demand for energy is smaller.

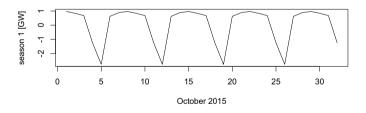


Figure 4. First seasonal component after the daily data decomposition. Source: Own study.

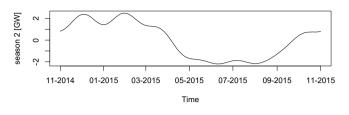


Figure 5. Second seasonal component after the daily data decomposition Source: Own study.

The analysis of the annual seasonality for daily data (Figure 3, Figure 5) allows to locate the drop in the maximum demand for power at the turn of the year and between June and August, which is in line with the seasonality observed in Figure 1. The annual cycle of changes in energy demand results mainly from changes in the day length and temperatures at different times of the year. In autumn and winter the sun rises later and sets earlier, which means more energy consumed by lighting. During this period, due to lower temperatures, additional power plants are activated in the energy system, where in addition to heat also electricity is produced.

Our results have shown, that TBATS (1 {5,5} 0.986 {<7.3> <365.25,7>}) is the most suitable to model daily data with double seasonality. Forecasts for the next year are shown in Figure 6 and Figure 7.

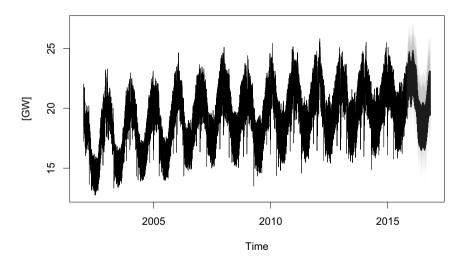
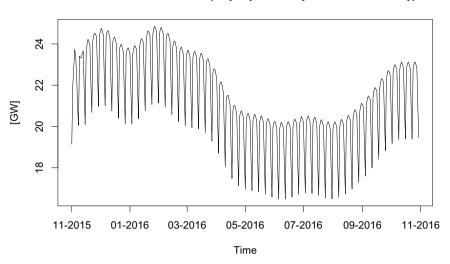


Figure 6. Historical data and the forecast of the maximum daily power demand in NPS in Poland until October 2016 Source: Own study.



Forecasts from TBATS(1, {5,5}, 0.986, {<7,3>, <365.25,7>})

Figure 7. Forecast of the maximum daily power demand in NPS in Poland until October 2016. Source: Own study.

In this model following parameters have been estimated: the Box-Cox transformation equals 1 (has no influence), errors ARMA(5,5), damping parameter equal 0.986 (has practically no influence), 3 pairs of Fourier series with the period  $m_1=7$  (weekly) and 7 pairs of Fourier series with the period  $m_2=365.25$  (annual). This model can be written as:

$$y_{t} = l_{t-1} + b_{t-1} + s_{t-1}^{(1)} + s_{t-1}^{(2)} + \alpha d_{t}$$

$$b_{t} = b_{t-1} + \beta d_{t}$$

$$s_{t}^{(1)} = \sum_{j=1}^{3} s_{j,t}^{(1)}$$

$$s_{j,t}^{(1)} = s_{j,t-1} \cos\left(\frac{2\pi jt}{7}\right) + s_{j,t-1}^{*} \sin\left(\frac{2\pi jt}{7}\right) + \gamma_{1}^{(1)} d_{t} .$$

$$s_{j,t}^{*(1)} = -s_{j,t-1} \sin\left(\frac{2\pi jt}{7}\right) + s_{j,t-1}^{*} \cos\left(\frac{2\pi jt}{7}\right) + \gamma_{2}^{(1)} d_{t} .$$

$$s_{t}^{(2)} = \sum_{j=1}^{7} s_{j,t}^{(2)}$$

$$s_{j,t}^{(2)} = s_{j,t-1} \cos\left(\frac{2\pi jt}{365.25}\right) + s_{j,t-1}^{*} \sin\left(\frac{2\pi jt}{365.25}\right) + \gamma_{1}^{(2)} d_{t}$$

$$s_{j,t}^{*(2)} = -s_{j,t-1}\sin\left(\frac{2\pi jt}{365.25}\right) + s_{j,t-1}^{*}\cos\left(\frac{2\pi jt}{365.25}\right) + \gamma_{2}^{(2)}d_{t}$$

re  $d_t$  is ARMA(5,5) process,  $\alpha$ ,  $\beta$ ,  $\gamma_1^{(1)}$ ,  $\gamma_2^{(2)}$ ,  $\gamma_1^{(2)}$  and  $\gamma_2^{(2)}$  are smoothing parameters. Weekly seasonality was approximately of 8 parameters (6 initial values for  $s^{(1)}_{j,0}$  and  $s^{*(1)}_{j,0}$  and two smoothing parameters  $\gamma_1^{(1)}$  and  $\gamma_2^{(1)}$ ), whereas the annual seasonality of 16 parameters (14 initial values for  $s^{(2)}_{j,0}$  and  $s^{*(2)}_{j,0}$  and two smoothing parameters  $\gamma_1^{(2)}$  and  $\gamma_2^{(2)}$ ). The total number of degrees of freedom is 32 (the remaining 8 come from two smoothing parameters  $\alpha$  and  $\beta$ , four parameters of ARMA process and the initial values and the slope values  $\ell_0$  and  $\delta_0$ .

The fitted above model is characterized by following error values:

 ME
 0,025845022

 RMSE
 0,758415990

 MAE
 0,423351049

 MPE
 -0,018148362 %

 MAPE
 2,235312999 %

 MASE
 0,301646593

 ACF1
 -0,006546378

Between MAE and RMSE errors there are no significant differences, the mean absolute percentage error (MAPE) is slightly over two percent, and MASE error points at a greater forecasting accuracy within the training set than the average naive forecast. These criteria and low autocorrelation (ACF1) between errors allow to evaluate the model as well adjusted to the data.

Forecasting this series with the use of conventional models such as ARIMA or exponential smoothing, it would be possible to determine forecasts for several observation forward only. If however we would like to create a forecast for several months, it would be necessary to group the data into monthly data, as shown in our earlier article (Brożyna, et al., 2016). Using TBATS model we can avoid these limitations and at the same time we can determine the medium-term forecast as detailed as the short-term forecasts. Figure 7 shows that the forecasts keep the weekly seasonality presented in Figure 4, that is the lower power demand at weekends, and the annual seasonality with a decline in demand at the turn of the year and during summer months as in Figure 5. Forecasts of daily data are characterized by a relatively narrow confidence intervals (Granger, 1980) averaging ±1.81 GW for the 80% confidence interval and ±2.77 GW for 95% confidence interval. Thus with the high probability we can assume that our results will be properly reflected in future energy demand. The advantages of the TBATS model translate directly into business, energy producers and the entire economy. With long, but detailed forecasts it is possible to minimize operating costs of energy producers and increase the energy security of the national economy.

# **CONCLUSIONS**

While analyzing time series containing daily data on power demand and performing its decomposition, we have detected, inter alia, that they are characterized by two seasonalities - annual and weekly. Forecasting of such data would not be possible using traditional models such as ARIMA or exponential smoothing,

therefore, we have presented an example of the use of more advanced model, such as TBATS, which allows to forecast data containing double seasonality and to specify the length of periods which took into account leap years. The accuracy of the obtained forecasts is confirmed both by the visual assessment and by low values of the calculated forecasts errors.

Using the TBATS model, unlike many others, it is possible to forecast the data with more than one seasonality, so the forecast horizon may include for example several months. Moreover the obtained forecast contain detailed information for each day. Such a combination of forecasting qualities can be important for the entire energy sector, not only because of their relevance, but also because it can address issues related to the economic aspects of security of the energy system. Detailed medium-term forecasts of energy demand can not only simplify the decision-making processes ranging from energy producers, and ending with the government, whose role is to take care of the general economy by negotiating international agreements, including the supply of energy sources and energy exchange with neighbouring countries, but it can also ensure the stability and continuity of the national power system.

In this article we have not explored all possibilities in terms of data frequency and number of seasonalities. In subsequent studies the analysis of hourly data in a short-term forecast will be performed, which would be particularly useful for energy producers, who design a production for a different times of a day and days of the week. It would be interesting to check the model performance based on an hourly data for a longer forecasting horizon, e.g. an annual forecast, which would include triple seasonality and give information about the energy demand for the particular hours, days and months.

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